Random Chess Thoughts

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Abstract

I dunno, random stuff.

1 Introduction

Chess is a game. A good game. A finite but uncountable game. The first two plys are unrestricted and then shit gets real. There are also a finite number of draw by insufficient material positions.

Here are a bunch of figures that I will need later, let's see where they go

2 First move

Convention dictates that white moves first. White has 20 legal first moves; 16 pawn moves and 4 Knight moves(see Appendix A). Following this black has the corresponding 20 legal moves, regardless of what white played. This gives a total of 400 possible first moves. Computing the number of possibilities for the second move is less straight forward, as it is unclear how many moves each player has. Depending on the previous moves, some pieces may not be restricted in their movements, possibly even unable to move. Additionally, pieces may have more moves than in the previous positions. We illustrate this by computing the number of possible moves in 3 different well known openings.

We start with the most well known, and indeed the most common, opening moves in chess, the King's Pawn Opening 1.e4 e5, which we show in Figure 3. It is easy to see that the e Pawn has no moves, as black's e Pawn blocks it. All other Pawns still have their original two moves, giving 14 possible Pawn moves. The Knights also both have their initial two moves each, giving a further 4 possible Knight moves. Finally, the King now has 1 move, giving us a total of $14 + 4 + 1 = 19$ possible moves, down from the initial 20.

Next, we look at an opening which doesn't reduce the number of possible moves white has, the Sicilian Opening 1.e4 c5, which we show in Figure 4. We see that the e pawn can move freely, but it now only has 1 moves, as compared to the 2 before. All other Pawns still have their original two moves, giving 15 possible moves. The Knights also both have their initial two moves each, giving a further 4 possible moves. Finally, the King now has 1 possible move. This gives us a total of $15 + 4 + 1 = 20$ possible moves, which is the same number as when we began.

Finally, we look at an opening, where white has more possible moves than at the beginning, the Scandinavian Opening 1.e4 d5, which we show in Figure 5. We see that the e pawn can move freely, but it also can capture black's d Pawn with exd5, giving it 2 possible moves. All other Pawns still have their original two moves, giving 16 possible moves. The Knights also both have their initial two moves each, giving a further 4 possible moves. Finally, the King now has 1 move, giving us a total of $16 + 4 + 1 = 21$ possible moves, up from the initial 20.

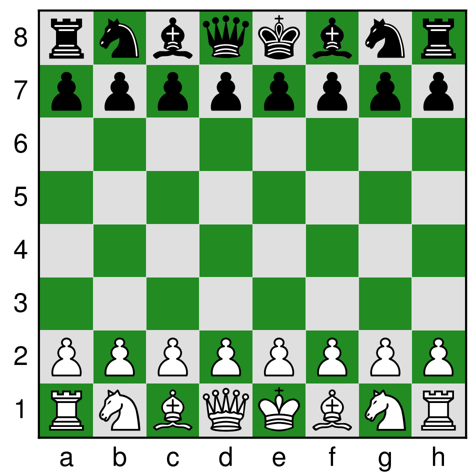

Figure 1: The starting position.

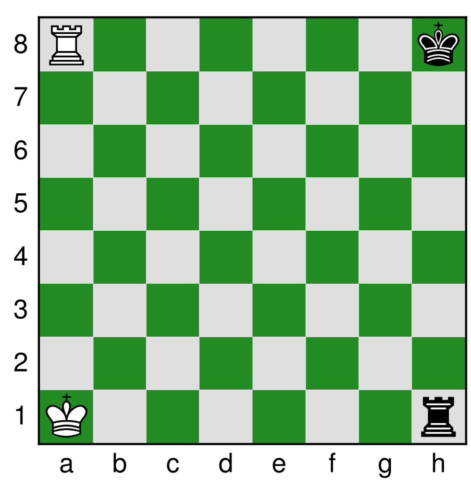

Figure 2: An illegal position, as both Kings are in check.

1 e4 e5

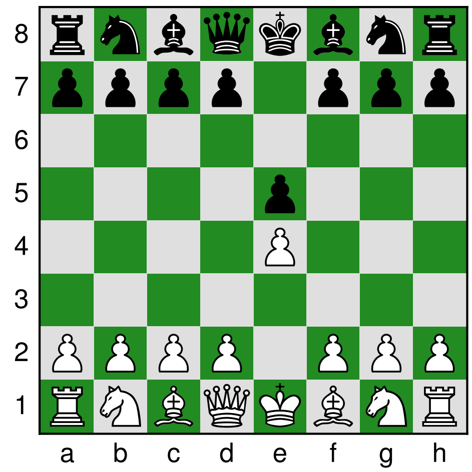

Figure 3: The King's Pawn Opening

1 e4 c5

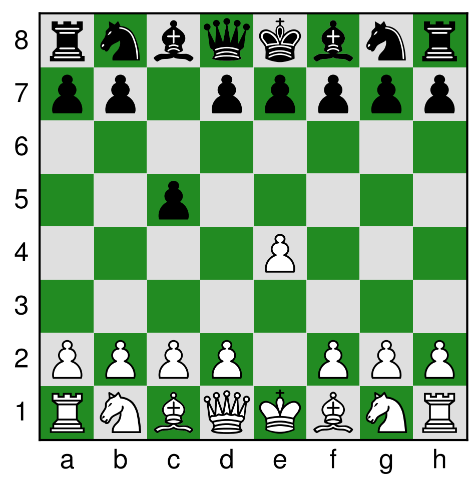

Figure 4: The Sicilian Opening

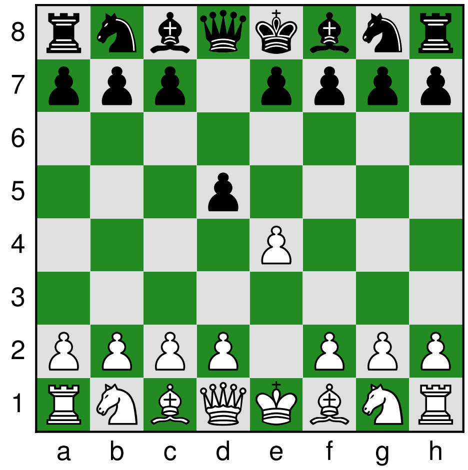

Figure 5: The Scandinavian Opening

3 Draw by Insufficient Material

We now all enumerate all the possible combinations for a draw by insufficient material. This occurs when neither player is capable of checkmating the other. This occurs in the following circumstances:

1. King vs King
2. King and Knight vs King
3. King and Bishop vs King
4. Kings with bishops of same colour
   (a) King and light-squared Bishop vs King and light-squared Bishop
   (b) King and dark-squared Bishop vs King and dark-squared Bishop

We examine each of the possibilities in order, as it allows us to systematically compute the total number of possibilities.

3.1 Bare Kings

We now enumerate all the *legal* configurations of two ♔s on a chessboard. Do do this we place the white ♔(wlog) and then count the number so squares the black ♔can occupy. We need only do this for one quadrant, as number of free spaces is symmetrical about the rank and the files.

We start by looking at possibilities in the first rank. We first place the white ♔at a1. By rules of chess, the black ♔cannot be next to the white ♔, hence it may not be in the squares a2, b1 or b2, as marked on the diagram below. This leaves $64 - 1 - 3 = \underline{60}$ possible placements for the black ♔.

Next we place white ♔on b1. We see that this blocks the black ♔from being on a1,a2,b2,c1 and c2, as marked inf Figure 8. This now leaves $64 - 1 - 5 = \underline{58}$ possible placements for the black ♔. We see similar arguments apply when the white ♔is on c1 (Figure 7) and d1 (Figure 9.

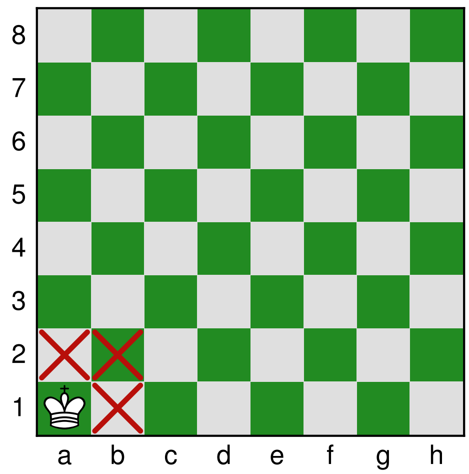

Figure 6: Positons where the black ♔may not by placed when the white ♔is on a1

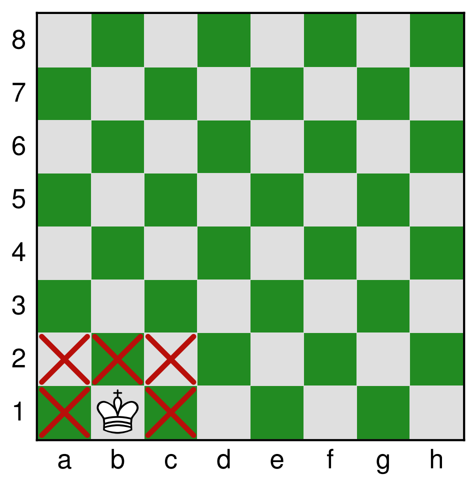

Figure 7: Positons where the black ♔may not by placed when the white ♔is on c1

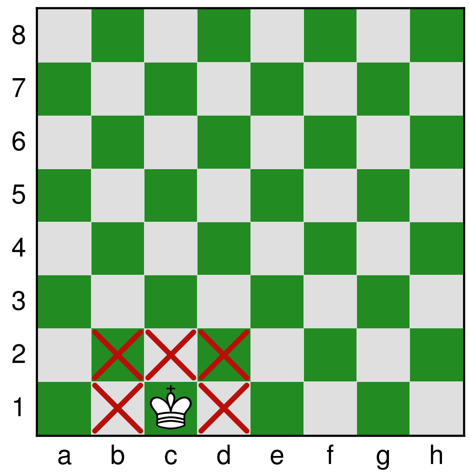

Figure 8: Positons where the black ♔may not by placed when the white ♔is on b1

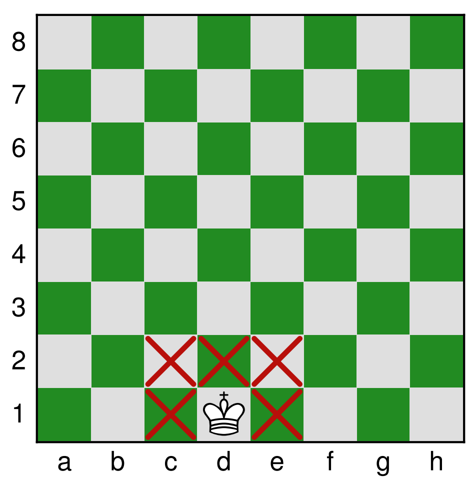

Figure 9: Positons where the black ♔may not by placed when the white ♔is on d1

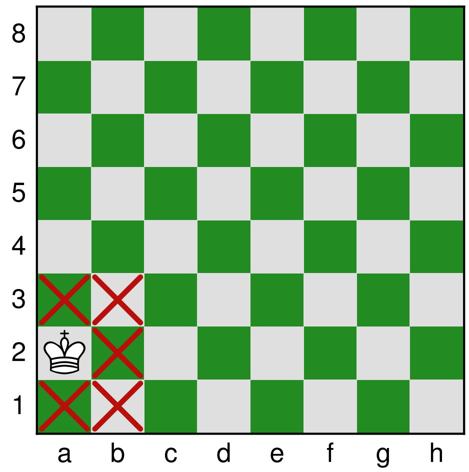

Figure 10: Positons where the black ♚may not by placed when the white ♔is on a2

We go no further along the first rank, as we simply need to compute the values for one quadrant. Now we look at the other possibilities along the a-file, which are also somewhat restricted. Having already checked a1, we place the white ♔on a2 and note that the black ♚cannot be on a1,a3,b1,b2 or b3, as marked in Figure 10. This now leaves $64 - 1 - 5 = \underline{58}$ possible placements for the black ♚. We see similar arguments apply when the white ♔is on a3 (Figure 11) and a4 Figure 12, both of which we show below.

Finally we look at what happens when we place the white ♔on any of the other squares in the quadrant. Since none of these squares are on the edge of the board, they each prevent the black ♚from occupying a the full 8 squares around it. Thus placing the white ♔on any of the squares b2,b3,b4,c2,c3,c4,d2,d3 or d4, leaves the black ♚$64 - 1 - 5 = \underline{55}$ possible locations. For completeness we give diagrams for these squares.

To provide an overveiw, we now show all of these positions, but we replace the ♔with the number of positions the black ♚when the white ♔is on that square. By simply adding up these numbers, we have to the total number of legal positions of two only two Kings being placed on the board. It is clear to see that this number is $(4 \times 60) + (24 \times 58) + (36 \times 55) = 240 + 1,392 + 1,980 = \underline{\mathbf{3,612}}$.

3.2 King and Knight vs King

We now compute how many possible arrangements there for the case where one player has a ♔and ♘and the other player only has a ♚. When adding any pieces to a chess position, we must consider if the placement forms a valid chess position or not. In particular, we need to check if the position results in an illegal check situation, e.g. when both Kings are in check(cf. Figure 2). However, it is clear to see that this will not be possible in this scenario. An interesting point to note is that some positions may result in a stalemate, which for our purposes is not a problem, as it would also result in a draw.

Consider the constructed example in Figure 14. After white has played Rc2+, black has two moves that will result in a draw. The first one Kxc2, as shown in Figure 15 will result in draw by insufficient material, but if black goes for Nxc2, Figure 16, this will cause a draw both by insufficient material and by stalemate. The white ♔cannot move to a2 and b1 as it would then be in check from the ♘and the black ♚prevents it from moving to b2. For the purposes of this section, we will simply ignore stalemates. We will also ignore if either ♔is in check, as this is immaterial for this discussion.

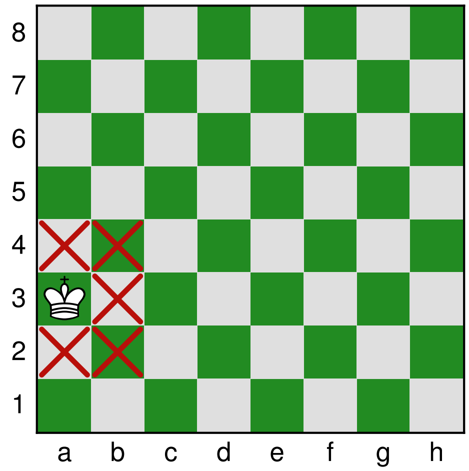

Figure 11: Positons where the black ♔may not by placed when the white ♔is on a3

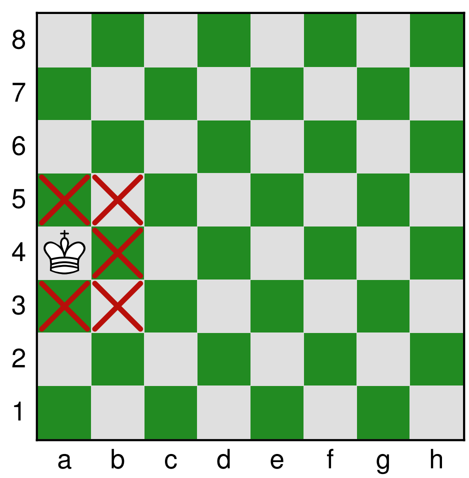

Figure 12: Positons where the black ♔may not by placed when the white ♔is on a4

	a	b	c	d	e	f	g	h
8	60	58	58	58	58	58	58	60
7	58	55	55	55	55	55	55	58
6	58	55	55	55	55	55	55	58
5	58	55	55	55	55	55	55	58
4	58	55	55	55	55	55	55	58
3	58	55	55	55	55	55	55	58
2	58	55	55	55	55	55	55	58
1	60	58	58	58	58	58	58	60

Figure 13: The number of possible placements for the black ♔with only the white ♔on the board.

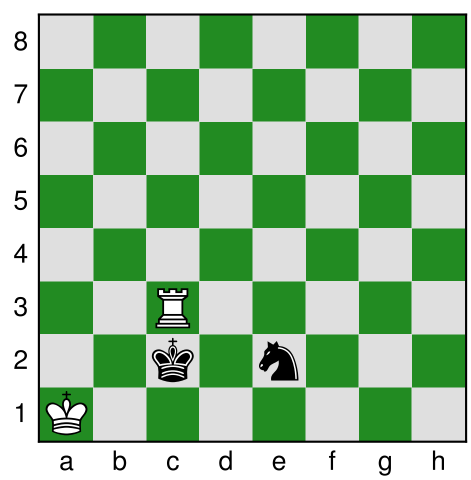

Figure 14: Constructed Stalemate Position #1

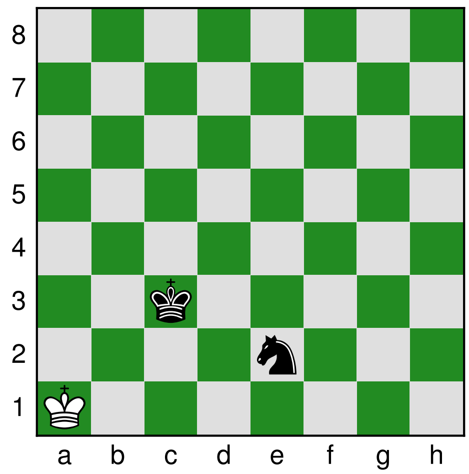

Figure 15: Constructed Stalemate Position #2

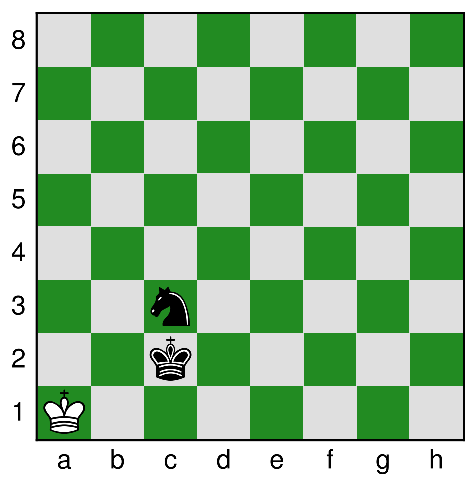

Figure 16: Constructed Stalemate Position #3

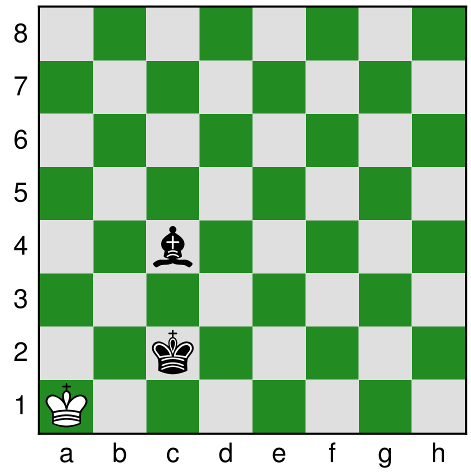

Figure 17: Constructed Stalemate Position #4

Now all that remains is to compute the number of possible positions. To do this, we first consider how many squares may a white ♘be placed on. It is clear to see that there is no restriction on where it may go, apart from that it may not occupy the same square as either ♔. Thus it is easy to see that we can place a white ♘on any one of $64 - 2 = 62$ squares. Combining this with our results from the previous section, this gives us $3,612 \times 62 = \underline{223,944}$ possible configurations of a white ♔and ♘and a black ♔. Clearly, the same arguments hold for the black ♘, thus we have $\underline{223,944}$ possible configurations of a white ♔and a black ♔and ♘. This gives us a total of $\underline{\mathbf{447,888}}$ positions of a ♔and ♘versus a ♔.

3.3 King and Bishop vs King

We now compute the number of possible positions where one player has a ♔and a ♗against the other player with just the king. While at first glance, it may seem that we need to take care when counting, this is not exactly true. If we were restricted to a specific type of ♗(light or dark square) then this would be relevant, however we can use any kind of ♗

Due to the nature of the Bishops, they can occupy only half of the squares on the board, either the light squares or the dark squares. The first idea is given an arbitrary legal positioning of the king, we count how many squares a light squared ♗can occupy and how many squares a dark squared ♗can occupy. The sum of these gives the total number of legal positions with a ♔and ♗vs a ♔.

However, we see that no matter which squares the Kings occupy, there are 32 squares an *arbitrary* bishop may occupy. Therefore, we get exactly the same arguments as those we used for the Knights in the previous section. Again we will not distinguish between position that are a draw by insufficient material and by stalemate, such as the position in Figure 17. By similar arguments as above, we have a total of $\underline{\mathbf{447,888}}$ positions of a ♔and ♗versus a ♔.

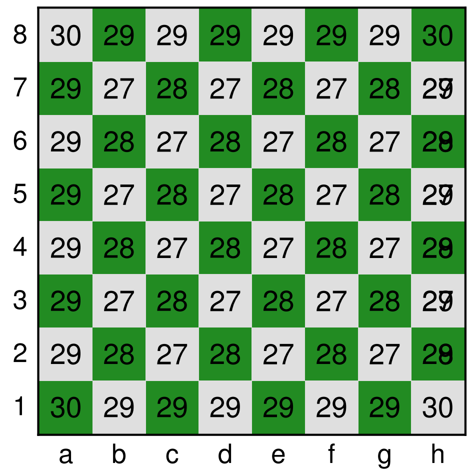

Figure 18: The number of possible light square placements for the black ♚with only the white ♔on the board.

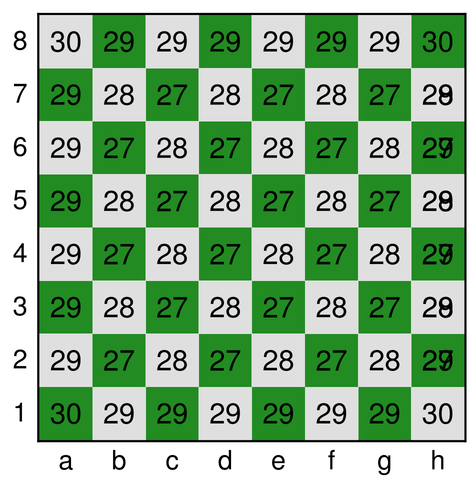

Figure 19: The number of possible light dark placements for the black ♚with only the white ♔on the board.

1 d4 e6 2 e4 d5 3 ♘c3 c5 4 ♘f3 ♘c6 5 e×d5 e×d5 6 ♗e2 ♘f6 7 O-O ♗e7 8 ♗g5 O-O 9 d×c5 ♗e6 10 ♘d4 ♗×c5 11 ♘×e6 f×e6 12 ♗g4 ♕d6 13 ♗h3 ♖ae8 14 ♕d2 ♗b4 15 ♗×f6 ♖×f6 16 ♖ad1 ♕c5 17 ♕e2 ♗×c3 18 b×c3 ♕×c3 19 ♖×d5 ♘d4 20 ♕h5 ♖ef8 21 ♖e5 ♖h6 22 ♕g5?? ♖×h3! 23 ♖c5 ♕g3!!

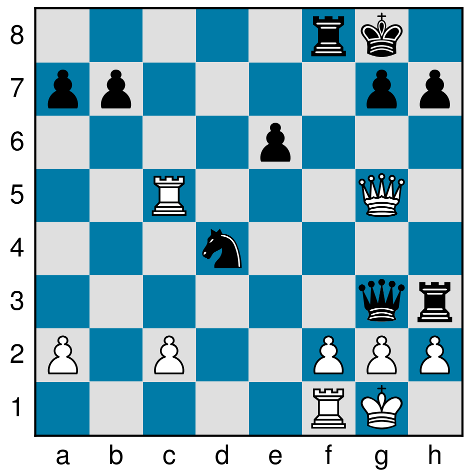

Figure 20: Levitsky v. Marshall June 10 1912 a.k.a The American Beauty.

References

A Ply 1

Here is an exhaustive list of all possible first moves for white.

B Ply 2

Here is an exhaustive list of all possible first moves for black, for compatactness, white is assumed to have moved 1.e4.

1 e4 e5 2 ♘f3 ♘c6 3 ♗c4 ♗c5 4 b4 ♗×b4 5 c3 ♗a5 6 d4 e×d4 7 O-O d3 8 ♕b3 ♕f6 9 e5 ♕g6 10 ♖e1! ♘ge7 11 ♗a3 b5?! 12 ♕×b5 ♖b8 13 ♕a4 ♗b6 14 ♘bd2 ♗b7? 15 ♘e4 ♕f5? 16 ♗×d3 ♕h5 17 ♘f6+!? g×f6 18 e×f6 ♖g8 19 ♖ad1! ♕×f3? 20 ♖7×e1+ ♘×e7? 21 ♕×d7+!! ♔×d7 22 ♗f5+ ♔e8 23 ♗d7+ ♔f8 24 ♗×e7#

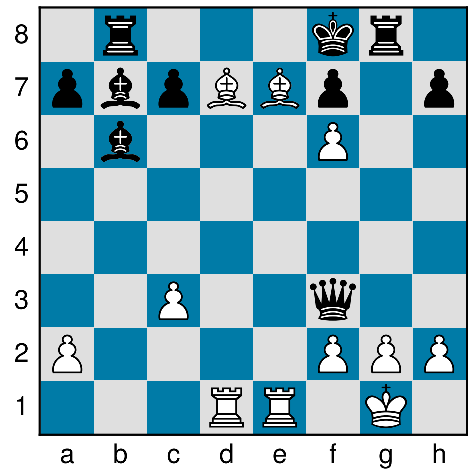

Figure 21: Anderssen v. Dufresne 1852 a.k.a The Evergreen Game.

1 e4 e5 2 ♘f3 d6 3 d4 ♗g4?! 4 d×e5 ♗×f3 5 ♕×f3 d×e5 6 ♗c4 ♘f6? 7 ♕b3 ♕e7 8 ♘c3 c6 9 ♗g5 b5? 10 ♘×b5! c×b5? 11 ♗×b5+ ♘bd7 12 O-O-O ♖d8 13 ♖×d7 ♖×d7 14 ♖d1 ♕e6 15 ♗×d7+ ♘×d7 16 ♕b8+! ♘×b8 17 ♖d8#

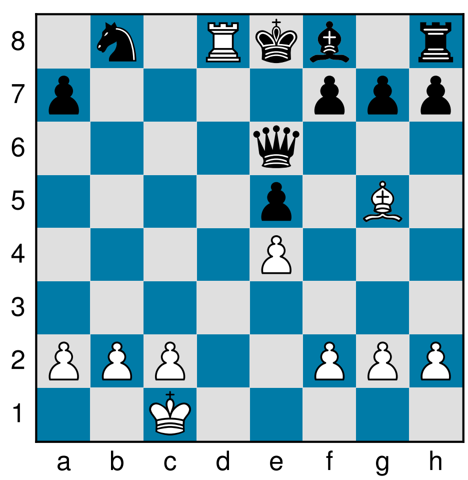

Figure 22: Morphy v. Duke Karl / Count Isouard, Paris 1858, a.k.a The Opera Game.

1 ♘f3 ♘f6 2 c4 g6 3 ♘c3 ♗g7 4 d4 O-O 5 ♗f4 d5 6 ♕b3 d×c4 7 ♕×c4 c6 8 e4 ♘bd7 9 ♖d1 ♘b6 10 ♕c5 ♗g4 11 ♗g5? ♘a4!! 12 ♕a3 ♘×c3 13 b×c3 ♘×e4! 14 ♗×e7 ♕b6 15 ♗c4 ♘×c3! 16 ♗c5 ♖fe8+ 17 ♔f1 ♗e6!! 18 ♗×b6? ♗×c4+ 19 ♔g1 ♘e2+ 20 ♔f1 ♘×d4+ 21 ♔g1 ♘e2+ 22 ♔f1 ♘c3+ 23 ♔g1 a×b6 24 ♕b4 ♖a4! 25 ♕×b6 ♘×d1 26 h3 ♖×a2 27 ♔h2 ♘×f2 28 ♖e1 ♖×e1 29 ♕d8+ ♗f8 30 ♘×e1 ♗d5 31 ♘f3 ♘e4 32 ♕b8 b5 33 h4 h5 34 ♘e5 ♔g7 35 ♔g1 ♗c5+ 36 ♔f1 ♘g3+ 37 ♔e1 ♗b4+ 38 ♔d1 ♗b3+ 39 ♔c1 ♘e2+ 40 ♔b1 ♘c3+ 41 ♔c1 ♖c2#

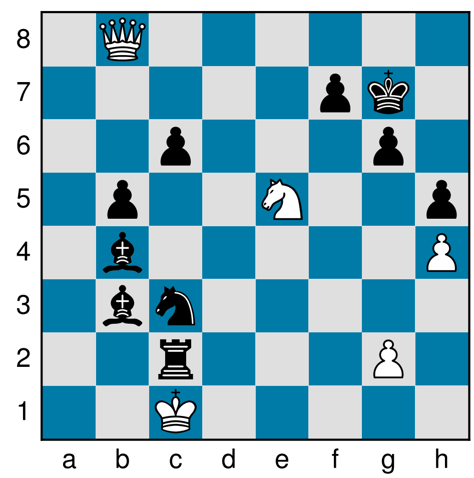

Figure 23: Byrne v. Fischer, New York, Oct. 17 1956 a.k.a The Game of the Century.